Quantenfeldtheorie

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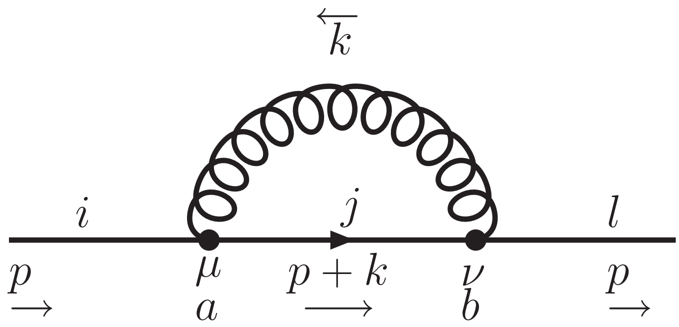
Aufgabe 10: *One-loop quark self-energy*

The one-loop correction for the quark self energy is given by the following Feynman diagram, denoted by $i\Sigma(\not{p}, m)$:

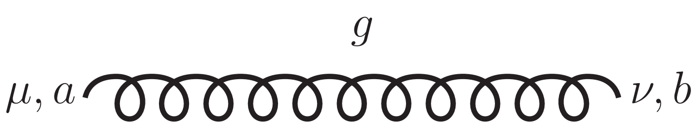


p and k denote the momenta, i, j and l denote the color of the quark, μ and ν are the usual Dirac indices and a and b denote different gluons.

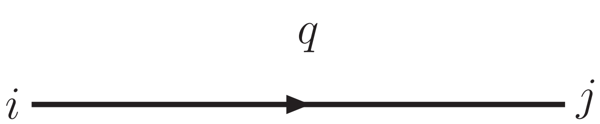
Evaluate $\Sigma(\not{p}, m)$ as far as possible in dimensional regularization:

a)

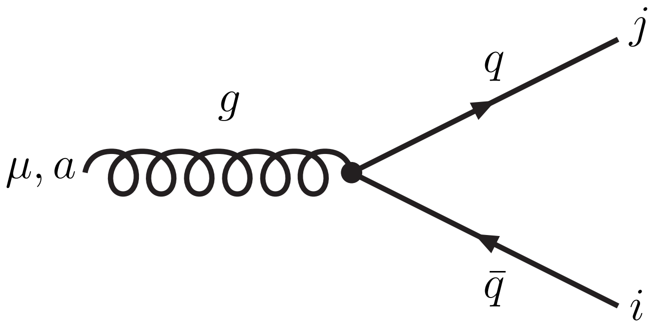
Use the following Feynman rules to express $\Sigma(\not{p}, m)$ (where p denotes the momentum of the propagating particle, its direction is from the left to the right):



$$-i\delta^{ab}\frac{g_{\mu\nu}}{p^2}$$



$$i\delta^{ij}\frac{\not{p}+m}{p^2-m^2}$$



$ig\gamma^\mu \left(T^a\right)_{ij}$

b)

Use dimensional regularization to evaluate the integral which appears.

The idea of dimensional regularization is to analytically continue integrals which diverge in four space-time dimension to a lower dimensionality D (where the integral converges). Before continuing the integral to D dimensions, one has to perform a 'Wick-rotation' however, in order to obtain an integral over Euclidean- rather than Minkowski space. To this end, one defines Eulidean momentum variables $k^0 \equiv ik_E^0, k^i \equiv k_E^i$.

One often defines $D = 4 - 2\epsilon$. In order for $\Sigma(\not{p}, m)$ to maintain the correct mass dimension, one also replaces $g \longrightarrow g\mu^\epsilon$.

Using these prescriptions and $(T^aT^a)_{il} = C_F\delta_{il}$, $C_F = 4/3$, you should then get

$$\Sigma(\not{p}, m) \quad = \quad -g^2 C_F \delta_{il} \mu^{2\epsilon} \int \frac{d^D k}{(2\pi)^D i} \frac{(2-D)\,(\not{p} + \not{k}) + Dm}{\left[(p+k)^2 - m^2\right] k^2} \,.$$

c)

Now apply the 'Feynman trick' to rewrite the propagators:

First, note the identiy

$$\frac{1}{A \cdot B} = \int_0^1 dx \frac{1}{\left[xA + (1-x)B\right]^2}.$$

Use this to rewrite

$$\frac{1}{\left[(p+k)^2 - m^2\right] k^2} = \int_0^1 \frac{dx}{\left[\tilde{k}^2 - M^2\right]^2} ,$$
$$\text{with} \qquad \tilde{k} = k + px, M^2 = x\left(m^2 - p^2(1-x)\right) .$$

Now perform the shift $k \to \tilde{k}$ for the integration variable. Notice that terms which are linear in the momentum k vanish after integration.

d)

Now, use (you may also show this, of course)

$$\int \frac{d^D k}{(2\pi)^D i} \frac{1}{\left[k^2 - M^2\right]^2} = \frac{1}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma\left(2 - \frac{D}{2}\right)}{\left(M^2\right)^{2-\frac{D}{2}}} .$$

to get

$$\Sigma(\not{p}, m) = -\frac{g^2 C_F \delta_{il}}{(4\pi)^2} \mu^{2\epsilon} \Gamma(\epsilon)(4\pi)^\epsilon \int_0^1 dx \frac{(2-D)\ \not{p}(1-x) + Dm}{\left(M^2\right)^\epsilon} .$$

d) As the last step, expand the previous result in ϵ. For simplicity, set $m = 0$ now.

First, perform the x-integration, making use of the identity

$$\int_0^1 x^{p-1}(1-x)^{q-1} dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} .$$

Then expand the result in ϵ, using

$$\Gamma(\epsilon) \approx \frac{1}{\epsilon} - \gamma_E .$$

You should end up with the final result

$$\Sigma(\not{p}, 0) = \frac{\alpha_s}{3\pi} \delta_{il}\ \not{p} \left[\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + \ln \frac{\mu^2}{-p^2} + 1 + \mathcal{O}(\epsilon)\right] .$$