Quantenfeldtheorie

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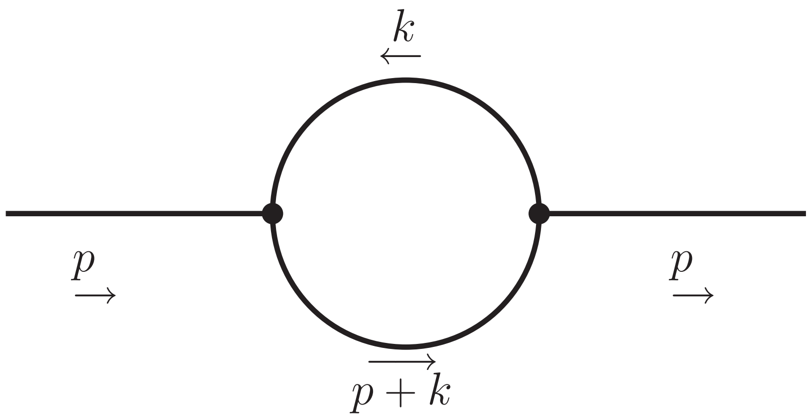
Aufgabe 10: *Calculation of the one-loop selfenergy for ϕ^3-theory in dimensional regularization and in lattice regularization*

Consider ϕ^3-theory given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3 \,. \tag{1}$$

a) The one-loop selfenergy is given by the following Feynman diagram, denoted by $-i\Sigma(p,m)$:



Using the Feynman rules that were given in the lecture, and using dimensional regularization (cf. exercise sheet 5), compute $\Sigma(p,m;\epsilon)$. You should end up with

$$\Sigma(p,m;\epsilon) = -\frac{g^2}{32\pi^2}\left\{\frac{1}{\epsilon} - \gamma - \int_0^1 dx \ln\left[\frac{m^2 - x(1-x)p^2}{4\pi\mu^2}\right]\right\} + \mathcal{O}(\epsilon)\,. \tag{2}$$

b) Dimensional regularization is only one of several possibilities to 'quantify' the divergence of divergent loop diagrams. Another, in some sense more intuitive, regularization scheme is lattice regularization. The basic idea is that one defines the quantum field theory on a lattice rather than on a continuous space-time. This cuts off the short distance divergencies.

Compute the one-loop selfenergy for ϕ^3-theory using lattice regularization:
As the first step, subract and add

$$\frac{ig^2}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-\mu^2]^2} \tag{3}$$

(where μ is an arbitrary mass scale) to the selfenergy, so that the selfenergy is split into a finite part and a divergent part.

Next, the divergent part is evaluated on a lattice with lattice spacing a. This means that high-momentum modes with momentum $q > 1/a$ do not exist. Consequently, the propagator is

$$S_F(q, m; a) = \frac{i}{q^2 - m^2} \quad \text{for} \quad qa \leqslant 1 \tag{4}$$

and zero else. Use this propagator to evaluate the divergent part of the selfenergy, Eq. (3). You should get

$$\Sigma_{div}(p, m; a) = -\frac{g^2}{16\pi^2} \ln(1/(a\mu)) + \text{finite}\,. \tag{5}$$

c) Which of the symmetries of the action are respected by dimensional- respectively lattice regularization?