Quantenfeldtheorie

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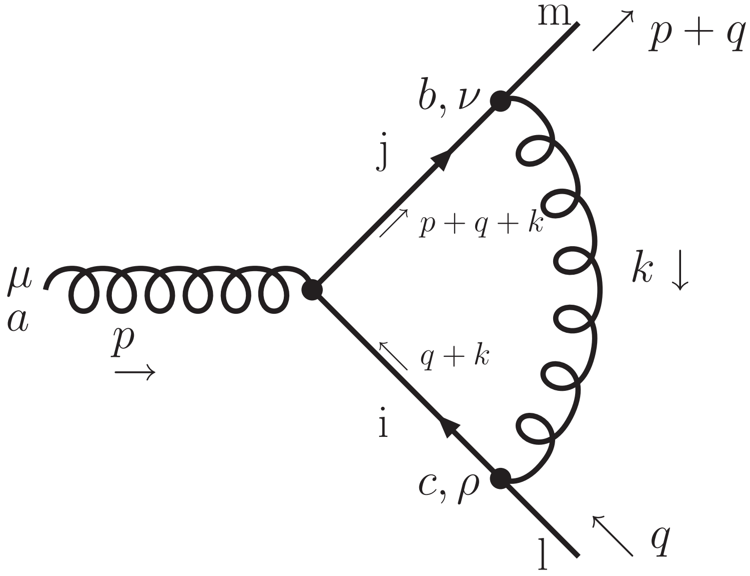
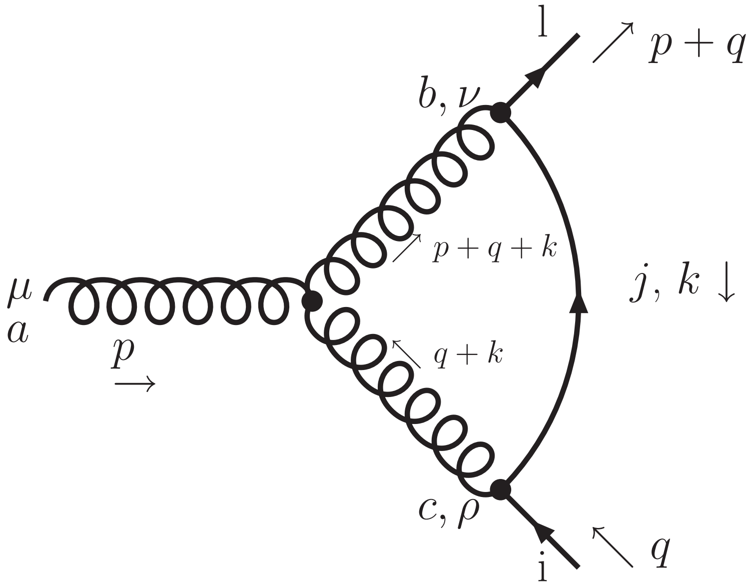
Aufgabe 19: *QCD vertex corrections*

The one-loop correction to the quark-gluon vertex is given by the following Feynman diagrams:



and



Using the Feynman rules from exercise sheet 10, calculate the UV-divergent part of these diagrams in dimensional regularization. You should find

$$\Gamma^\mu(p,q,m)|_{UV} = -\frac{1}{6} ig T^a_{lm} \gamma^\mu \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}$$

for the first diagram and

$$\Gamma^\mu(p,q,m)|_{UV} = \frac{9}{2} ig T^a_{il} \gamma^\mu \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}$$

for the second diagram.

Hint: This calculation can be done essentially analogously to that for the quark- and gluon self-energies. In order to simplify the computation, one can also use a trick however, as explained in the following:

In contrast to the quark and gluon self-energies, the superficial degree of divergence of the one-loop vertex corrections is zero. Therefore, one may extract the UV-divergent part of the above diagrams by setting the mass and the external momenta to zero. (To understand this point, read chapter 10.1 in the QFT book by Peskin and Schroeder.) The expression which you then get is not only UV-divergent, but also has an (unphysical) IR divergence, because the mass and external momenta were set to zero. You have to be careful to only extract the UV-divergent part, for instance by rewriting

$$\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2)^2} = \lim_{M\to 0}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{(k^2-M^2)^2}$$

and then dropping the IR-divergent terms which arise in the limit $M \to 0$.

Aufgabe 20: *The β function for QCD*

You have now calculated all diagrams which are needed to determine the one-loop β function for QCD. We summarize here these results (only the divergent parts):

$$\begin{aligned}
i\Sigma_{il}(\not p, m)|_{UV} &= \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{4}{3}\cdot i\delta_{il}\,(\not p - 4m)\\
i\Pi^{q,ab}_{\mu\nu}(p,m)|_{UV} &= \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{-2}{3}\cdot i\delta^{ab}\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right)\\
i\Pi^{g,ab}_{\mu\nu}(p,m)|_{UV} &= \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot 5\cdot i\delta^{ab}\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right)\\
\Gamma^\mu(p,q,m)|_{UV} &= \frac{1}{\epsilon}\cdot\frac{\alpha_s}{4\pi}\cdot\frac{13}{3}\cdot ig\gamma^\mu T^a\,.
\end{aligned}$$

Use this to find

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(11 - \frac{4}{3}n_F\right)\,,$$

where n_F is the number of families.